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INFLUENCE OF NONUNIFORMITY OF THE TEMPERATURE DISTRIBUTION IN THE KNUDSEN LAYER ON THE VELOCITY OF THERMAL SLIP OF A RAREFIED GAS ALONG A CURVED SURFACE

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The correction to the velocity of thermal slip of a rarefied gas along the surface of a sphere has been calculated; the correction allows for the presence of the second-order mixed derivative of temperature. For this purpose we constructed an exact solution of the Boltzmann inhomogeneous kinetic equation with the collision operator in the form of an ellipsoidal statistical model. The results obtained confirm the existence of negative (in the direction of the temperature gradient) thermophoresis of highly heat-conducting aerosol particles at small values of the Knudsen number. Comparison with the literature data has been made.

Exact analytical solutions of an ellipsoidal statistical (ES) model of the Boltzmann kinetic equation in problems of thermal and isothermal slips of rarefied gas along the surfaces of a sphere and a right circular cylinder are obtained in [1–3]. In what follows, using the ES model of the Boltzmann kinetic equation we solve the problem of calculation of the correction to the velocity of thermal slip of a rarefied gas along a spherical surface; the correction allows for the presence of the mixed second-order derivative of temperature. Allowance for this effect made it possible to predict the existence of negative (in the direction of the temperature gradient) thermophoresis of highly heat-conducting aerosol particles.

Problem Formulation. Derivation of Basic Equations. We consider a rarefied gas which fills the space around a spherical aerosol particle of radius R. The state of the gas is described by the distribution function $f(\mathbf{r}, \mathbf{C})$, which is the solution of the linearized Boltzmann kinetic equation with the collision operator in the form of the ES model [4, 5]. In the spherical system of coordinates this equation is written as

$$C_r \frac{\partial f}{\partial r} + \frac{1}{r}\left[C_\theta \frac{\partial f}{\partial \theta} + \frac{C_\varphi}{\sin\theta}\frac{\partial f}{\partial \varphi} + (C_\theta^2 + C_\varphi^2)\frac{\partial f}{\partial C_r} + (C_\varphi^2 \operatorname{ctan}\theta - C_r C_\theta)\frac{\partial f}{\partial C_\theta} - (C_\varphi C_\theta \operatorname{ctan}\theta + C_r C_\varphi)\frac{\partial f}{\partial C_\varphi}\right] =$$

$$= f^0(\mathbf{r}, \mathbf{C})\left[1 + \beta^{-3/2}\iiint K(\mathbf{C}, \mathbf{C}')\, f(\mathbf{r}, \mathbf{C}')\, d\mathbf{C}'\right] - f(\mathbf{r}, \mathbf{C}) ,$$

$$K(\mathbf{C}, \mathbf{C}') = 1 + 2\mathbf{C}\mathbf{C}' + \frac{2}{3}\left(C^2 - \frac{3}{2}\right)\left(C'^2 - \frac{3}{2}\right) - 2C_iC_j\left(C_i'C_j' - \frac{1}{3}\delta_{ij}C'^2\right).$$

Let the temperature gradient which is normal to the particle surface be set in the gas. We assume that the gradient is not constant but slowly changes along the particle surface. Thus, in the problem the quantities $\partial T/\partial r$ and $\partial^2 T/\partial r\partial\theta$ differ from zero. The first of these quantities leads to a temperature jump on the particle surface and the other to additional slip of the gas along its surface, which is caused by the nonuniformity of the temperature distribution in the Knudsen layer. We assume these quantities to be small, i.e., we take that

$$\frac{1}{T_s}\left|\frac{\partial T}{\partial r}\right| << 1 , \quad \frac{1}{T_s}\left|\frac{\partial^2 T}{\partial r\partial\theta}\right| << 1 .$$

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We linearize the distribution function of gas particles over the coordinates and velocities relative to the locally equilibrium distribution function written in the Chapman–Enskog approximation [6]:

$$f = f^0 \, [1 + Y(r, \theta, \mathbf{C})] \, .$$

As the boundary condition on the particle surface we take the model of diffuse reflection.

We expand $Y(r, \theta, \mathbf{C})$ into a series in terms of a small parameter R^{-1}:

$$Y(r, \theta, \mathbf{C}) = Y^{(1)}(r, \theta, \mathbf{C}) + R^{-1} Y^{(2)}(r, \theta, \mathbf{C}) + \dots \, .$$

Substituting this series into the initial equation and equating the terms at R^{-1}, we come to the equation for the determination of $Y^{(2)}(r, \theta, \mathbf{C})$

$$C_r \frac{\partial Y^{(2)}}{\partial r} + Y^{(2)}(r, \theta, \mathbf{C}) = \pi^{-3/2} \iiint \exp(-C'^2) \, K(\mathbf{C}, \mathbf{C}') \, Y^{(2)}(r, \theta, \mathbf{C}') \, d\mathbf{C}' -$$

$$- \left[(C_\theta^2 + C_\varphi^2) \frac{\partial Y^{(1)}}{\partial C_r} + (C_\varphi^2 \operatorname{ctan}\theta - C_r C_\theta) \frac{\partial Y^{(1)}}{\partial C_\theta} - (C_\varphi C_\theta \operatorname{ctan}\theta + C_r C_\varphi) \frac{\partial Y^{(1)}}{\partial C_\varphi} \right] - C_\theta \frac{\partial Y^{(1)}}{\partial \theta} \qquad (1)$$

with the boundary conditions

$$Y^{(2)}(r, \theta, \mathbf{C}) = -2 C_\theta U_\theta^2 \big|_s \, , \quad C_r > 0 \, ; \quad Y^{(2)}(\infty, \theta, \mathbf{C}) = 0 \, . \qquad (2)$$

It follows from the problem of the temperature jump at the boundary of the solid surface [7] that

$$Y^{(1)}(r, \theta, \mathbf{C}) = Y_1(r, \theta, C_r) + (C_\theta^2 + C_\varphi^2 - 1) \, Y_2(r, \theta, C_r) \, . \qquad (3)$$

We find the solution of Eq. (1) in the form

$$Y^{(2)}(r, \theta, \mathbf{C}) = C_\theta \varphi(r, \theta, C_r) + \sum_k b_k(C_\theta, C_\varphi) \, Y_k^{(2)}(r, \theta, C_r) \, , \qquad (4)$$

where C_θ in combination with $b_k(C_\theta, C_\varphi)$ forms a complete system of orthogonal (in the meaning of the scalar product) polynomials.

We substitute (4) into (1) and (2). Remultiplying the obtained relations by $C_\theta \exp(-C_\theta^2 - C_\varphi^2)$ and integrating with respect to C_θ and C_φ from $-\infty$ to $+\infty$, with account for (3) we come to the equation and the boundary conditions for determining the function $\varphi(r, \theta, C_r)$:

$$L\varphi(x, \mu) = -k \, [Y_1(x, \mu) + Y_2(x, \mu)] \, , \qquad (5)$$

$$\varphi(0, \mu) = -2U_0 \, , \quad \mu > 0 \, , \qquad (6)$$

$$\varphi(\infty, \mu) = 0 \, . \qquad (7)$$

Here

$$k = \frac{1}{T_s} \frac{\partial^2 T}{\partial r \partial \theta} \, , \quad Y(x, \mu) = \int_0^\infty \exp(-x/\eta) \, F(\eta, \mu) \, A(\eta) \, d\eta \, , \quad \mu = C_r \, ,$$

$$x = r - R\,, \quad F(\eta, \mu) = \frac{1}{\sqrt{\pi}}\,\eta\, P\,\frac{1}{\eta - \mu}\,\Delta(\mu) + \exp(\eta^2)\,\Lambda(\eta)\,\delta(\eta - \mu)\,,$$

$$\Lambda(z) = \lambda(z)\,\Delta(z) + M\,, \quad \lambda(z) = 1 + \frac{1}{\sqrt{\pi}}\, z \int_{-\infty}^{\infty} \frac{\exp(-\mu^2)\, d\mu}{\mu - z}\,,$$

$$\Delta(\mu) = \begin{bmatrix} 1 & \mu^2 - 1/2 \\ -1/2 & 1/2 \end{bmatrix}, \quad M = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad U_0 = U_\theta^{(2)}\big|_s\,,$$

$$A(\mu) = [A_1(\mu), A_2(\mu)]^t\,, \quad Y(x, \mu) = [Y_1(x, \mu), Y_2(x, \mu)]^t\,.$$

For brevity, the argument θ is omitted in the functions entering into (5), and the integro-differential operator corresponding to the ES model of the Boltzmann kinetic equation is symbolized by $L\varphi(x, \mu)$, i.e.,

$$L\varphi(x, \mu) \equiv \mu \frac{\partial \varphi}{\partial x} + \varphi(x, \mu) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-\tau^2)\,\varphi(x, \tau)\, d\tau + \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} \tau \exp(-\tau^2)\,\varphi(x, \tau)\, d\tau\,.$$

We take

$$a(\eta, \mu) = \left[\frac{1}{2} A_1(\eta) + \mu^2 A_2(\eta)\right] k\,, \quad b(\mu) = \left\{ a(\mu, \mu)\,\lambda(\mu) + \left[\frac{1}{2} A_1(\mu) + \mu^2 A_2(\mu)\right] k \right\} \exp(\mu^2)\,.$$

Then, with account for the adopted designation we rewrite (5) in the form

$$L\varphi(x, \mu) = -\int_0^{\infty} \exp(-x/\eta)\,\frac{\eta a(\eta, \mu)}{\eta - \mu}\, d\eta - \exp(-x/\mu)\, b(\mu)\,\Theta_+(\mu)\,. \qquad (8)$$

Thus, the problem of calculation of the correction to the velocity of thermal slip of a rarefied gas along a spherical surface, which allows for the presence of the second-order mixed derivative of temperature, is reduced to solution of Eq. (8) with boundary conditions (6) and (7).

Problem Solution. We find a general solution of the inhomogeneous integro-differential equation (8) in the form of the sum of the general solution of the corresponding homogeneous equation and a partial solution of the inhomogeneous one.

Direct substitution ensures that

$$\varphi_0(x, \mu) = A_0 + \int_0^{\infty} \exp(-x/\eta)\,\Phi(\eta, \mu)\, n(\eta)\, d\eta$$

is the general solution of the corresponding homogeneous equation. Here

$$\Phi(\eta, \mu) = \frac{1}{\sqrt{\pi}}\,\eta P\,\frac{1}{\eta - \mu} + \exp(\eta^2)\,\lambda(\eta)\,\delta(\eta - \mu)\,,$$

and A_0 and $n(\eta)$ are the unknown parameters to be determined.

We find a partial solution of the equation

$$L\varphi_1(x, \mu) = -\frac{1}{\sqrt{\pi}} \int_0^{\infty} \exp(-x/\eta)\,\frac{\eta a(\eta, \mu)}{\eta - \mu}\, d\eta \qquad (9)$$

in the form

$$\varphi_1(x,\mu) = -x\int_0^\infty \exp(-x/\eta)\,\Phi(\eta,\mu)\,[a(\eta,\eta)/\eta]\,d\eta +$$

$$+\frac{x}{\mu}\exp(\mu^2 - x/\mu)\,\lambda(\mu)\,a(\mu,\mu)\,\Theta_+(\mu) + \int_0^\infty \exp(-x/\eta)\,G_1(\eta,\mu)\,d\eta \tag{10}$$

provided that

$$\int_{-\infty}^{\infty} \exp(-\mu^2)\,G_1(\eta,\mu)\,d\mu = 0\,. \tag{11}$$

Substituting (10) to (9), we come to the characteristic equation

$$\left(1-\frac{\mu}{\eta}\right)G_1(\eta,\mu) = \frac{1}{\sqrt{\pi}}\left[-\frac{1}{2}A_1(\eta) + \mu\eta A_2(\eta) - (\eta\lambda(\eta)\,a(\eta,\eta))' + \mu\eta\lambda(\eta)\,a(\eta,\eta) - \frac{1}{2}\mu A_1(\eta)\right],$$

whose solution in the space of generalized functions has the form

$$G_1(\eta,\mu) = \frac{1}{\sqrt{\pi}}\eta P\frac{1}{\eta-\mu}\left[-\frac{1}{2}A_1(\eta) + \mu\eta A_2(\eta) - (\eta\lambda(\eta)\,a(\eta,\eta))' + \right.$$

$$\left. + \mu\eta\lambda(\eta)\,a(\eta,\eta) - \frac{1}{2}\mu A_1(\eta)\right] + g_1(\eta)\,\delta(\eta-\mu)\,, \tag{12}$$

where $g_1(\eta)$ is found by substitution of (12) into the normalization condition (11):

$$g_1(\eta) = \exp(\eta^2)\left[m(\eta,\eta)\,\lambda(\eta) + \frac{1}{2}A_1(\eta) + (\eta\lambda(\eta)\,a(\eta,\eta))'\right].$$

Here

$$m(\eta,\mu) = -\frac{1}{2}[\mu\eta+1]\,A_1(\eta) + \mu\eta A_2(\eta) - (\eta\lambda(\eta)\,a(\eta,\eta))' + \mu\eta\lambda(\eta)\,a(\eta,\eta)\,.$$

The partial solution of the equation

$$L\varphi_2(x,\mu) = -\exp(-x/\mu)\,b(\mu)\,\Theta_+(\mu)$$

is found in a similar way:

$$\varphi_2(x,\mu) = -\frac{x}{\mu}\exp(-x/\mu)\,b(\mu)\,\Theta_+(\mu) + \int_0^\infty \exp(-x/\eta)\,G_2(\eta,\mu)\,d\eta\,,$$

$$G_2(\eta,\mu) = \frac{1}{\sqrt{\pi}}\eta P\frac{1}{\eta-\mu}\left([\eta b(\eta)\exp(-\eta^2)]' - \mu\eta\,b(\eta)\exp(-\eta^2)\right) + g_2(\eta)\,\delta(\eta-\mu)\,,$$

$$g_2(\eta) = \exp(\eta^2)\,[(\eta b(\eta)\exp(-\eta^2))'(\lambda(\eta)-1) - \eta^2\,b(\eta)\exp(-\eta^2)]\,.$$

With account for the results obtained we write the general solution of Eq. (8) in the form

$$\varphi(x,\mu) = A_0 + \int_0^\infty \exp(-x/\eta)\,\Phi(\eta,\mu)\left[n(\eta) - \frac{x}{\eta}\,a(\eta,\eta)\right]d\eta +$$

$$+ \int_0^\infty \exp(-x/\eta)\,[G_1(\eta,\mu) + G_2(\eta,\mu)]\,d\eta + \frac{x}{\mu}\exp(-x/\mu)\,[\exp(\mu^2)\,\lambda(\mu)\,a(\mu,\mu) - b(\mu)]\,\Theta_+(\mu)\,. \quad (13)$$

The unknown parameters A_0 and $n(\eta)$ entering into (13) are determined from the boundary conditions (6), (7) by the boundary-value problem theory.

Determination of the Parameters Entering into the Solution. The solution constructed at $A_0 = 0$ satisfies the boundary condition (7) at infinity. With account for the boundary condition (6) on the wall, we reduce (13) to a singular integral equation with the Cauchy kernel

$$-2U_0 = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{\eta n(\eta)\,d\eta}{\eta-\mu} + \exp(\mu^2)\,\lambda(\mu)\,n(\mu) + \exp(\mu^2)\,a_4(\mu) +$$

$$+ \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{\eta a_3(\eta,\mu)\,d\eta}{\eta-\mu} + \exp(\mu^2)\,\lambda(\eta)\,a_3(\mu)\,, \quad \mu > 0\,, \quad (14)$$

where

$$a_3(\eta,\mu) = A_2(\eta) - \mu\eta A_1(\eta) + \eta\left[\frac{1}{2}A_1(\eta) + A_2(\eta)\right]'; \quad a_4(\mu) = -A_2(\mu) - \mu\left[\frac{1}{2}A_1(\mu) + A_2(\mu)\right]'.$$

We denote

$$M(z) = \frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{\eta a_3(\eta,z)\,d\eta}{\eta - z}$$

and introduce the auxiliary function

$$N(z) = \frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{\eta n(\eta)}{\eta - z}\,d\eta\,, \quad (15)$$

which is analytic in the complex plane with a cut along the real positive semiaxis. With allowance for the form of the boundary values of the functions $N(\mu)$, $M(\mu)$, and $\lambda(\mu)$ from above and below on the cut $(0, +\infty)$, we reduce Eq. (14) to the Riemann inhomogeneous boundary-value problem on the real positive semiaxes:

$$[N^+(\mu) + M^+(\mu) + U_0]\,\lambda^+(\mu) - [N^-(\mu) + M^-(\mu) + U_0]\,\lambda^-(\mu) = -\sqrt{\pi}\,i\mu a_4(\mu)\,, \quad \mu > 0\,. \quad (16)$$

We consider the corresponding homogeneous boundary-value problem

$$\frac{X^+(\mu)}{X^-(\mu)} = \frac{\lambda^+(\mu)}{\lambda^-(\mu)}\,, \quad \mu > 0\,.$$

As the zero-bounded solution of the problem we take the function

$$X(z) = \frac{1}{z} \exp\left(\frac{1}{\pi} \int_0^\infty \frac{\zeta(\tau)}{\tau - z} d\tau \right), \quad \zeta(\tau) = -\pi/2 - \arctan \frac{\lambda(\tau)}{\sqrt{\pi}\, \tau \exp(-\tau^2)} .$$

Using the considered homogeneous problem, we reduce (16) to the problem of determination of the analytical function by the prescribed jump

$$[N^+(\mu) + M^+(\mu) + U_0]\, X^+(\mu) - [N^-(\mu) + M^-(\mu) + U_0]\, X^-(\mu) = -\sqrt{\pi}\, i\mu a_4(\mu) \frac{X^-(\mu)}{\lambda^-(\mu)}, \quad \mu > 0 ,$$

whose solution with account for the behavior of all the functions entering into it is written in the form

$$N(z) = -M(z) - U_0 - \frac{1}{2\sqrt{\pi}} \frac{1}{X(z)} \int_0^\infty \frac{X^-(\mu)}{\lambda^-(\mu)} \frac{\mu a_4(\mu)\, d\mu}{\mu - z} . \tag{17}$$

The function $N(z)$ determined by equality (15) disappears at an infinitely remote point. We require that solution (17) also possess this property. Expanding (17) into a series in the vicinity of the infinitely remote point, we find

$$U_0 = -\frac{1}{2\sqrt{\pi}} \left[\int_0^\infty \eta^2 A_1(\eta)\, d\eta - \int_0^\infty \frac{X^-(\mu)}{\lambda^-(\mu)} \mu a_4(\mu)\, d\mu \right].$$

Hence, after double integration by parts we have

$$U_0 = -\frac{k}{2\sqrt{\pi}} \left[\int_0^\infty \eta^2 A_1(\eta)\, d\eta - \int_0^\infty \frac{\eta\, [3A_1(\eta) + 4A_2(\eta)]\, d\eta}{X(-\eta)} - \frac{1}{\pi} \int_0^\infty \frac{\eta^2\, [A_1(\eta) + 2A_2(\eta)]\, d\eta}{X(-\eta)} \int_0^\infty \frac{\zeta(\tau)\, d\tau}{(\tau + \eta)^2} \right]. \tag{18}$$

Expression (18) determined the sought-for correction to the velocity of thermal slip, which allows for the presence of the second-order mixed derivative of temperature. The numerical analysis presented gives the following result:

$$U_0 = 0.4150885k . \tag{19}$$

Passing in (19) to dimensional quantities and writing in the form adopted in the kinetic theory of rarefied gases [8]

$$U_\theta \big|_s = K_T\, \beta_R\, \mathrm{Kn}\, \nu k ,$$

we find β_R = 1.834375. Allowing for the fact that for highly heat-conducting aerosol particles at small values of the Knudsen number, the rate of thermophoresis is determined by the expression [9]

$$\mathbf{U}_T = \tau\nu\, \mathrm{Kn}\, \nabla \ln T , \quad \tau = -2K_T (C_T + \beta_R - \beta_{\mathrm{Ba}}) ,$$

where K_T = 1.14995, C_T = 2.204939, and β_{Ba} = 4.297, we find τ = 0.5926. We note that the value of the coefficient β_R provides theoretical confirmation of the existence of negative (in the direction of the temperature gradient) thermophoresis. Application of the method used in the present paper to the BGK model of the Boltzmann kinetic equation with constant and variable collision frequencies gives values of the coefficient β_R equal to 2.376842 and 1.1437, respectively.

Thus, in the paper, we calculated the correction to the velocity of thermal slip of a rarefied gas along the surface of a spherical aerosol particle; the correction allows for the presence of the second-order mixed derivative of temperature. It is shown that allowance for the considered effect in the boundary conditions provides theoretical confirmation of the existence of negative (in the direction of the temperature gradient) thermophoresis for highly heat-conducting particles at small values of the Knudsen number. The results obtained can be used for calculating the thermophoresis rate of aerosol particles in temperature-nonuniform rarefied gases. The analysis conducted allows one to draw a conclusion on the substantial dependence of the value of the coefficient β_R on the choice of the model of the collision integral.

NOTATION

R, dimensionless sphere radius; $3R\lambda/\sqrt{\pi}$, dimensional sphere radius, m; λ, mean free path of gas particles, m; $\mathrm{Kn} = \sqrt{\pi}/3R$, Knudsen number; $\mathbf{C}$, dimensionless velocity of gas molecules; $\mathbf{C}\beta^{-1/2}$, dimensional velocity of gas molecules, m/sec; $\beta = m/2k_BT$, Boltzmann constant; m, mass of a molecule, kg; $U_\theta|_s$, value of the tangent to the surface of the component of the dimensionless mass velocity on the particle surface; T, temperature, K; T_s, particle surface temperature, K; K_T, coefficient of thermal slip of rarefied gas along a plane surface; k, second-order mixed derivative of temperature related to the temperature of the particle surface; $f(\mathbf{r}, \mathbf{C})$, distribution function of gas molecules over the coordinates and velocities; $f^0(\mathbf{r}, \mathbf{C}) = (\beta/\pi)^{3/2} \exp(-C^2)$, absolute Maxwellian; $f^0(\mathbf{r}, \mathbf{C})$, locally equilibrium distribution function of gas molecules in the gas volume written in Chapman–Enskog approximation; C, modulus of the dimensionless velocity of gas molecules; C_r, C_θ, C_φ, components of the dimensionless intrinsic velocity of molecules in the spherical coordinate system; r, modulus of the dimensionless radius-vector; $3r\lambda/\sqrt{\pi}$, dimensional radius-vector, m; θ, φ, angular coordinates of the spherical coordinate system; ν, kinematic viscosity of gas, m^2/sec; x, dimensionless distance reckoned from the normal to the sphere surface; $Y(r, \theta, \mathbf{C})$, function allowing for deviation of the distribution function in the Knudsen layer from the distribution function in the gas volume; $Y_1(x, \mu)$ and $Y_2(x, \mu)$, distribution functions obtained in the problem of a temperature jump; μ, radial component of the dimensionless velocity of gas molecules; η, spectral parameter of expansion; $\Phi(\eta, \mu)$, eigenvectors of the continuous spectrum of the problem of isothermal slip of a rarefied gas along a plane surface (Kramers problem); $F(\eta, \mu)$, eigenvectors of the continuous spectrum of the problem of a temperature jump; $\lambda(z)$, Cercignani dispersion function; a, auxiliary function; L, linearized operator; τ, coefficient; Px^{-1}, distribution in the meaning of the main value in calculation of the integral of x^{-1}; $\delta(x)$, Dirac delta function; $\Theta_+(\mu)$, Heaviside step function ($\Theta_+(\mu) = 0$ when $\mu < 0$ and $\Theta_+(\mu) = 1$ when $\mu \geq 0$); i, imaginary unit; $X(z)$, canonical function from the Kramers problem; C_T, coefficient of the temperature jump of rarefied gas at the boundary of the plane surface; β_{Ba}, coefficient of Barnett slip; β_R, coefficient allowing for the effect of nonuniformity of the temperature distribution in the Knudsen layer on the velocity of slip. Indices: s, sphere surface; r, θ, φ, projections on the axes of the spherical coordinate system; B, Boltzmann; Ba, Barnett; T, temperature; 0, zero; $'$, differentiation; (1), (2), ordinal number of the corresponding coefficients in expansion of quantities into a series in terms of powers of the inverse radius; $\pm$, denotes boundary values of the functions of the complex variable on the upper and lower edges of the cut; i, j, projections of velocity on the coordinate axes of the spherical coordinate system; t, transposition.

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